

# Erratum to “Feasible sequential quadratic programming for finely discretized problems from SIP”

Craig T. Lawrence and André L. Tits  
*Department of Electrical Engineering  
and Institute for Systems Research  
University of Maryland, College Park  
College Park, Maryland 20742*

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## Abstract

Two errors are pointed out in the convergence analysis in the quoted paper [Semi-Infinite Programming, Kluwer Academic Publishers, 1998, pp. 159–193]. Correct proofs are provided.

Below, two errors are pointed out in the convergence analysis in the quoted paper [1]. Correct proofs are provided. Note that these proofs only apply when  $d^1(x, \hat{\Xi})$  is defined to be the solution of  $QP^1(x, \hat{\Xi})$ .

In the proof of Lemma 3.5(iii), Lemmas 3.1 and 3.2 are both applied without first showing that  $\Xi_{\text{act}}(x^*) \subseteq \Xi^*$ . We offer here an alternative proof which does not rely on establishing this fact.

**Lemma 3.5(iii):**  $t_k d_k \rightarrow 0$ .

*Proof.* In view of non-positivity of the optimal values for the QPs defining  $d_k^0$  and  $d_k^1$ , the inequalities

$$\begin{aligned}\langle \nabla f(x_k), d_k^0 \rangle &\leq -\frac{1}{2} \langle d_k^0, H_k d_k^0 \rangle, \\ \langle \nabla f(x_k), d_k^1 \rangle &\leq -\frac{1}{2} \langle d_k^1, d_k^1 \rangle\end{aligned}$$

hold. Thus,

$$\begin{aligned}\langle \nabla f(x_k), d_k \rangle &= (1 - \rho_k) \langle \nabla f(x_k), d_k^0 \rangle + \rho_k \langle \nabla f(x_k), d_k^1 \rangle \\ &\leq -\frac{(1 - \rho_k)}{2} \langle d_k^0, H_k d_k^0 \rangle - \frac{\rho_k}{2} \langle d_k^1, d_k^1 \rangle.\end{aligned}$$

In view of the line search criterion

$$\begin{aligned}f(x_k) - f(x_{k+1}) &\geq -\alpha t_k \langle \nabla f(x_k), d_k \rangle \\ &\geq \frac{\alpha t_k (1 - \rho_k)}{2} \langle d_k^0, H_k d_k^0 \rangle + \frac{\alpha t_k \rho_k}{2} \langle d_k^1, d_k^1 \rangle \\ &\geq 0.\end{aligned}$$

Since we have established that the sequence  $\{f(x_k)\}$  converges, and since both terms on the right-hand side of the second line are positive, they must both converge to zero. Further, note that the inequalities still hold if the first term on the right-hand side of the second line is multiplied by  $t_k(1 - \rho_k) \in [0, 1]$  and the second term is multiplied by  $t_k \rho_k \in [0, 1]$  (since both terms are positive). As  $H_k$  is positive definite for all  $k$  and bounded away from singularity, this gives

$$\begin{aligned}t_k(1 - \rho_k)d_k^0 &\rightarrow 0, \\ t_k \rho_k d_k^1 &\rightarrow 0.\end{aligned}$$

Adding these shows  $t_k d_k \rightarrow 0$ .  $\square$

In the proof of Proposition 3.7 in the appendix, in the second paragraph, the logic used to establish that  $d^{0,*}$  and  $d^{1,*}$  are both non-zero is flawed. In particular, again it has not been established that  $\Xi_{\text{act}}(x^*) \subseteq \Xi'$ , hence Lemmas 3.1 and 3.2 may not be applied. Note that under the contradiction assumption of Proposition 3.7,  $v^{0,*} + v^{1,*} > 0$ . Thus, in view of Lemma 3.6(i) and (ii), we cannot have *both*  $d^{0,*} = 0$  and  $d^{1,*} = 0$ , i.e. at least one must be non-zero. This, coupled with the following claim, gives the result we need.

**Claim:** Given  $x \in X$ ,  $H = H^T > 0$ , and  $\Xi' \subseteq \Xi$ ,  $d^0(x, H, \Xi') = 0$  if, and only if,  $d^1(x, \Xi') = 0$ .

*Proof.* Suppose  $d^0 = d^0(x, \Xi') = 0$  and let  $\lambda_\xi^0$ ,  $\xi \in \Xi'$ , denote the multipliers from  $QP^0(x, H, \Xi')$ . From the optimality conditions (2.2) for  $QP^0(x, H, \Xi')$  and (2.3) for  $QP^1(x, \Xi')$  it is not difficult to see that  $(d^1, \gamma) = (0, 0)$  is a

KKT point for  $QP^1(x, \Xi')$  with multipliers

$$\mu^1 = \frac{1}{1 + \sum_{\xi \in \Xi'} \lambda_\xi^0},$$

$$\lambda_\xi^1 = \frac{\lambda_\xi^0}{1 + \sum_{\xi \in \Xi'} \lambda_\xi^0}, \quad \xi \in \Xi'.$$

Uniqueness of such points establishes  $d^1(x, \Xi') = 0$ .

Now suppose  $d^1 = d^1(x, \Xi') = 0$ , thus  $\gamma(x, \Xi') = 0$ , and let  $\mu^1$  and  $\lambda_\xi^1$ ,  $\xi \in \Xi'$ , denote the multipliers from  $QP^1(x, \Xi')$ . As  $(d^1, \gamma) = (0, 0)$ , it follows from the constraints in  $QP^1(x, \Xi')$  that  $\Xi^1 \subseteq \Xi_{\text{act}}(x)$ , where  $\Xi^1 \subseteq \Xi'$  is the active set from  $QP^1(x, \Xi')$ . This implies  $\mu^1 > 0$ , otherwise the first equation in the optimality conditions (2.3) would violate Assumption 2. It then follows from the optimality conditions (2.2) and (2.3) that  $d^0 = 0$  is KKT for  $QP^0(x, H, \Xi')$  with multipliers

$$\lambda_\xi^0 = \frac{\lambda_\xi^1}{\mu^1}, \quad \xi \in \Xi'.$$

Uniqueness of such points establishes  $d^0(x, H, \Xi') = 0$ . □

## References

- [1] C. T. Lawrence and A. L. Tits. Feasible sequential quadratic programming for finely discretized problems from SIP. In R. Reemtsen and J.-J. Rückmann, editors, *Semi-infinite Programming*, pages 159–193. Kluwer Academic Publishers B.V., 1998. In the series Nonconvex Optimization and its Applications.