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ERRATUM TO "AN SQP ALGORITHM FOR FINELY DISCRETIZED CONTINUOUS MINIMAX PROBLEMS AND OTHER MINIMAX PROBLEMS WITH MANY OBJECTIVE FUNCTIONS"*

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Abstract. An error is pointed out in the local convergence proof in the quoted paper [*SIAM J. Optimization* 6(1996), 461]. A correct proof is given.

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The proof of Lemma 3.14 in [2] is incorrect. Namely, proving the claim in the second sentence of the proof does not "complete the proof" as stated. To see this, note that the mathematical induction argument hinted at in that sentence merely proves that, for the infinite index set K whose existence is established by Lemma 3.12, given any integer i, there exists k_i such that $\Omega_{max}(x^*) \subseteq \Omega^b_{k+i}$ for all $k \in K$, $k \geq k_i$. Since $\{k_i : i = 1, 2, ...\}$ is not shown to be bounded, it does not follow that $\Omega_{max}(x^*) \subseteq \Omega^b_k$ for all k large enough.

Nevertheless, the convergence results claimed in that section of [2] do hold true. A correct proof is obtained if Lemmas 3.13, 3.14, and 3.15 in [2] are replaced with Lemmas 3.13', 3.14', and 3.15' given below. (The statements of Lemmas 3.14' and 3.15' are identical to those of Lemmas 3.14 and 3.15, respectively, but the proofs are different.) The implicit assumptions required for the original Lemmas 3.13–3.15 are still assumed to hold.

LEMMA 3.13'. *Let $\sigma_1, \sigma_2 > 0$ be given and let*

$$\mathcal{H} = \{H = H^T : \sigma_1\|d\|^2 \leq \langle d, Hd\rangle \leq \sigma_2\|d\|^2 \quad \forall d \in I\!R^n\}.$$

Then, for every $\epsilon > 0$ there exists $\delta > 0$ such that for every x with $\|x - x^*\| < \delta$, every $H \in \mathcal{H}$, and every $\hat{\Omega} \subseteq \Omega$ with $\Omega_{max}(x^*) \subseteq \hat{\Omega}$, all $\omega \in \Omega_{max}(x^*)$ are binding for $QP(x, H, \hat{\Omega})$ and the unique KKT point d of $QP(x, H, \hat{\Omega})$ satisfies $\|d\| < \epsilon$.

Proof. Given x with $\|x - x^*\| < \delta$, $H \in \mathcal{H}$, and $\hat{\Omega} \subseteq \Omega$ with $\Omega_{max}(x^*) \subseteq \hat{\Omega}$, let $d(x, H, \hat{\Omega})$ be the unique KKT point of $QP(x, H, \hat{\Omega})$. Then Lemma 3.2 implies that $d(x^*, H, \hat{\Omega}) = 0$ for all $H \in \mathcal{H}$. Since $\mathcal{H}$ is compact, in view of Assumptions 4 and 5, it follows from a classical result of Robinson's [1, Theorem 2.1] that, given $\epsilon > 0$, there exists $\delta_{\hat{\Omega}} > 0$ such that, for all x with $\|x - x^*\| < \delta_{\hat{\Omega}}$ and all $H \in \mathcal{H}$, all $\omega \in \Omega_{max}(x^*)$ are binding for $QP(x, H, \hat{\Omega})$ and $\|d(x, H, \hat{\Omega})\| < \epsilon$. That $\delta_{\hat{\Omega}} > 0$ can be chosen independent of $\hat{\Omega}$ follows from finiteness of Ω. □

LEMMA 3.14'. *For k large enough, $\Omega_{max}(x^*) \subseteq \Omega^b_k$.*

Proof. Let $\sigma_1, \sigma_2 > 0$ be as given by Assumption 3, and let $\delta > 0$ be as given by Lemma 3.13' (for an arbitrary $\epsilon > 0$). Since $x_k \to x^*$ as $k \to \infty$ (Proposition 3.11) it follows from Lemma 3.12 that there exists $\hat{k}$ such that $\|x_k - x^*\| < \delta$ for all $k \geq \hat{k}$

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and $\Omega_{max}(x^*) \subseteq \Omega_{\hat{k}}$. That $\Omega_{max}(x^*) \subseteq \Omega_k$ for all $k \geq \hat{k}$ can now be proved by mathematical induction. Indeed, let $k \geq \hat{k}$ and suppose $\Omega_{max}(x^*) \subseteq \Omega_k$. It follows from Lemma 3.13' and Assumption 3 that $\Omega_{max}(x^*) \subseteq \Omega_k^b$ and, since, by construction, $\Omega_k^b \subseteq \Omega_{k+1}$ for all k, it follows that $\Omega_{max}(x^*) \subseteq \Omega_{k+1}$. That $\Omega_{max}(x^*) \subseteq \Omega_k^b$ for all $k \geq \hat{k}$ now directly follows from Lemma 3.13' and Assumption 3. □

LEMMA 3.15'. *The entire sequence* $\{d_k\}$ *converges to zero.*

Proof. Directly follows from Lemmas 3.13' and 3.14', Assumption 3, and the fact that, by construction, $\Omega_k^b \subseteq \Omega_{k+1}$ for all k. □

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REFERENCES

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